Student Edition TWA - A10

More Factoring

Difference of Two Squares

I solved 5^2 this way: 4 x 6 = 24 and 24 + 1 = 25.

I solved 8^2 this way: 7 x 9 = 63 and 63 + 1 = 64.

I solved 10^2 this way: 9 x 11 = 99 and 99 + 1 = 100.

I solved 20^2 this way: 19 x 21 = 399 and 399 + 1 = 400.

Will this method always work? Can you express it mathematically?

Does $n^2 = (n - 1)(n + 1) + 1$? Multiplying $(n - 1)(n + 1)$ gives $n^2 + n - n + 1 = n^2 - 1$. Adding the 1 gives $n^2 - 1 + 1 = n^2$

Can you say this in words? One less than the number times one more than the number is 1 less than the number squared. We can write this algebraically as $(n - 1)(n + 1) = n^2 - 1$.

It turns out that $10^2 - 2^2 = 12$ x 8 = 96, two more than the number times two less than the number.

It turns out that $10^2 - 3^2 = 13$ x 7 = 91, three more than the number times three less than the number.

We might generalize this to the difference of any two squares is the product of the square root of the first number plus the square root of the second times the first square root minus the second square root.

We can extend this to factoring the difference of two squares.

$x^2 - y^2 = (x + y)(x - y)$

Some other examples:

$9x^2 - 16 = (3x + 4)(3x - 4)$ $25r^2 - 1 = (5r + 1)(5r - 1)$ $100a^2 - 9b^2 = (10a + 3b)(10a - 3b)$

If the negative is first, try reversing the order:

$-49 + x^2$ to $x^2 - 49$ gives $(x - 7)(x + 7)$

Sometimes we can factor a polynomial with four terms into the product of two binomials.	$10x^2 + 15x + 8xy + 12y$
We can group the first two terms together and factor.	$10x^2 + 15x = (2x + 3)5x$
Likewise, we can group the last two terms and factor.	$8xy + 12y = (2x + 3)4y$
Now here is a significant leap in thinking. We can factor out a binomial 2x + 3 from both expressions leaving	$(2x + 3)(5x + 4y)$

Another example:	$28a^2 - 21ab - 8a + 6b$
Grouping we get:	$(28a^2 - 21ab) + (-8a + 6b)$
Factoring each expression, we get:	$(4a - 3b)7a + (-4a + 3b)2$
The terms inside the binomials are not the same, and we cannot factor yet, but they only differ by the signs of the terms. Instead of factoring out a 2 in the last group, we could instead factor out a -2.	$(4a - 3b)7a + (4a - 3b)(-2)$
Now, we can factor out the expression $4a - 3b$.	$(4a - 3b)(7a - 2)$

The previous factoring problem have all been purposely selected, but the first rule in factoring is to factor out any common factors from all terms and then continue factoring.

For example:	$5x^2 + 35x + 60$
We can factor out a 5 from each term giving:	$5(x^2 + 7x + 12)$
Then, we can factor the trinomial, which we have done before into:	$5(x + 3)(x + 4)$

For the example :	$6x^2 - 150$
We can first factor out a 6 giving:	$6(x^2 - 25)$
Since the binomial is the difference of squares, we can factor it into:	$6(x - 5)(x + 5)$

There are more strategies to factoring which are best understood through practice and experience. The prior examples were purposely selected to show different factoring strategies. Not all trinomials factor with integer coefficients. For example:

	$x^2 + x + 3$
Trying to factor we get:	$(x+\quad)(x+\quad)$
Since the only factors of 3 are 1 and 3, we try:	$(x + 1)(x + 3)$
Expanding this using the distributive property, we get:	$x^2 + 4x + 3$

which is not the same expression. There are no whole number solutions for this example.

This problem does have a solution but not by the factoring methods we have used. We will need another strategy called the quadratic formula. That will be one of the last things we do in this class.

Problem Set - A10

Multiply.

1. $(5x + 3)(5x – 3)$ 2. $(5x + 3)(5x + 3)$ 3. $(5x – 3)(5x -3)$

4. What similarities and differences do you see in the three solutions above?

5. Explain why the following mathematical expressions are not equivalent.

   $2yz – 2y \neq z$

6. A compass and a ruler cost 4 dollars. The compass costs 90 cents more than the ruler. How much does each cost?

Factor:

7. $x^2 – 121$ 8. $2x^2 – 288$ 9. $9y^2 – 64$

10. Why is 145 – 72 + 72 = 145? Can you express the idea in the equation using variables?

11. If x = 4, what is $3x^2 – 7x + 9$?

12. The problem above can be expressed as: $f(4) = 3x^2 – 7x + 9$, which simply means replace x with 4 and find the answer.

    Find $f(4)$ if $f(x) = 3x^2 - 7x + 9$.

Factor

13. $6x^2 + 5x - 21$

14. $2a^2 + 14ab + 5ac + 35bc$

15. The football team has the ball on the 29-yard line. There is a 15-yard penalty. Where could the ball be spotted after the penalty?

16. Solve the following problem mentally:

$$(867 \times 25) \times 4 = \_\_\_\_\_$$

17. Rhonda cannot remember how much money she had when she went to the mall. She spent half of the money on a shirt and then spent two-thirds of what was left to buy a gift. If she now has $1.35, how much did Rhonda begin with?

Factor each trinomial if you can. One cannot be factored.

18. $x^2 + 13x + 36$ 19. $x^2 - 5x - 7$ 20. $42x^2 + 11x - 3$

21. Which ordered pairs are solutions of the equation $3x + 2y = 36$?

a. (4,12) b. (8,6) c. (5,10)