# TWA - A10

## More Factoring

I solved $5^2$ this way 4 x 6 = 24 and
24 + 1 = 25.

I solved $8^2$ this way 7 x 9 = 63 and 63 + 1 = 64.

I solved $10^2$ this way 9 x 11 = 99 and 99 + 1 = 100.

I solved $20^2$ this way 19 x 21 = 399 and 399 + 1 = 400.

Will this method always work? Can you express it mathematically?

Does $n^2 = (n – 1)(n + 1) + 1$? Multiplying $(n – 1)(n + 1)$ gives $n^2 + n – n + 1 = n^2 – 1$ and then adding the 1 gives $n^2 – 1 + 1 = n^2$

Can you say this in words? One less than the number times one more than the number is 1 less than the number squared. We can write this algebraically as $(n – 1)(n + 1) = n^2 – 1$.

It turns out that $10^2 – 2^2 = 12 \times 8 = 96$, two more than the number times two less than the number.

It turns out that $10^2 – 3^2 = 13 \times 7 = 91$, three more than the number times three less than the number.

We might generalize this to the difference of any two squares is the product of the square root of first number plus the square root of second times the first square root minus the second square root.

We can extend this to factoring the difference of two squares.

$x^2 – y^2 = (x + y)(x – y)$

Some other examples:

$9x^2 – 16 = (3x + 4)(3x – 4)$   $25r^2 - 1 = (5r + 1)(5r – 1)$   $100a^2 – 9b^2 = (10a + 3b)(10a – 3b)$

If the negative is first try reversing the order:

$-49 + x^2$ to $x^2 – 49$ which gives $(x – 7)(x + 7)$

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| | Factoring four terms gets at the structure of algebra. The structure of algebra is a major theme of this project. In this example, students need to see 2x + 3 as both a process and an object. In order to factor it out, they need to see it as an object. Conceptually, this is no small feat. Talk about this with students. |
| | Some students do not take out a common factor when factoring a trinomial. |
| | Students should recognize that not all trinomials factor. |

Sometimes we can factor a polynomial with four terms into the product of two binomials.

If you want to factor $10x^2 + 15x + 8xy + 12y$, you can group the first two terms together andfactor. Likewise, you can group the last two terms together and factor:

$10x^2 + 15x = (2x + 3)5x$ and $8xy + 12y = (2x + 3)4y$. Notice that $2x + 3$ is a common factor for each term. You can factor out a binomial $2x + 3$ from both expressions leaving $(2x + 3)(5x + 4y)$

Another example:

$28a^2 – 21ab – 8a + 6b$ grouping we get $(28a^2 – 21ab) + (– 8a + 6b)$ factoring each expression weget $(4a – 3b)7a$ and $(-4a + 3b)2$ the terms inside the binomials are not the same and we cannot factor yet. But they only differ by the signs of the terms instead of factoring out a 2 in the last two terms we could instead factor out a -2 giving $(4a – 3b)7a$ and $(4a – 3b)(-2)$ now we can factor out the expression $4a – 3b$ giving us $28a^2 – 21ab – 8a + 6b = (4a – 3b)(7a – 2)$

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The previous factoring problems have all been purposely selected, but it can be helpful to factorout any common factors from all terms and then continue strategies for factoring. For example:

If you want to factor $5x^2 + 35x + 60$, you can factor out a 5 from each term giving $5(x^2 + 7x +12)$ and then you can factor the trinomial into $5(x + 3)(x + 4)$.

For $6x^2 – 150$ you can first factor out a 6 giving $6(x^2 – 25)$. Since the binomial is the differenceof square you can factor it into $6(x – 5)(x + 5)$

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There are more strategies to factoring which are best understood through practice and experience. The prior examples were purposely selected to show different factoring strategies.Not all trinomials factor with integer coefficients. For example;

If you want to factor $x^2 + x + 3$, you would start with $(x + \ )(x + \ )$. Since the only integer factors of 3 are 1 and 3 we have to use these numbers, plugging them in we get $(x + 3)(x + 1)$ and usingFOIL we get $x^2 + 4x + 3$ which is not the same as the original equation. There are no solutions with integers for the numbers for this example.

This problem can be factored, but not with integer coefficients and not by the factoring methodswe have used. There is another factoring strategy called the quadratic formula. This will be one of the last things we learn about in this class.

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| | 1-3. Multiplying binomials. |
| | 4. The key question is: can students first notice some of the similarities and differences AND use them to more efficiently solve these types of problems? |
| | 5. A non-example. This is not how to apply the distributive property. Non-examples are important in students' learning. They learn from both examples and non-examples! |
| | 6. Students could solve this problem using general problem solving strategies or algebra. Either way is okay! |
| | 7-9. Factoring the difference of squares. |

# Problem Set A10

Multiply

1. $(5x + 3)(5x - 3)$ 2. $(5x + 3)(5x + 3)$ 3. $(5x - 3)(5x - 3)$

$25x^2 - 9$ $25x^2 + 30x + 9$ $25x^2 - 30x + 9$

4. What similarities and differences do you see in the three solutions above?

Answers may vary. In #1, the middle terms drop out. In #2 and #3, the digits are thesame, but the sign is different for the middle term.

5. Explain why the following mathematical expressions are not equivalent. $2yz - 2y \neq z$

$2y(z - 1) \neq z$. You cannot subtract 2y from 2yz., You can factor out 2y as shown!

6. A compass and a ruler cost 4 dollars. The compass costs 90 cents more than the ruler.How much does each cost?

Compa

ss =

$2.45

Ruler =

$1.55

Factor:

7. $x^2 - 121$ 8. $2x^2 - 288$ 9. $9y^2 - 64$

$(x + 11)(x - 11)$ $2(x - 12)(x + 12)$ $(3y + 8)(3y - 8)$

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| | 10. Discuss how the associative property can take precedence over the order of operations in this problem. Also, you might discuss how you can think of - 72 as + (-72). |
| | 11. Substituting values into an expression is an important skill. |
| | 12. This problem is related to function notation. |
| | 13&14. Factoring four terms. The mathematics behind this process is particularly important. Discussing the solutions with students can help them understand this significant aspect of algebra. |

10. Why is $145 - 72 + 72 = 145$? Can you express the idea in the equation using variables? $145 - 72 + 72 = 145$ because $-72 + 72 = 0$ so all that is left is 145.

$$-x + x = 0$$

11. If $x = 4$ what is $3x^2 - 7x + 9$ equal to?

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12. The problem above can be expressed as:

$f(4) = 3x^2 - 7x + 9$ which simply means replace x with 4 and find the answer.

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Factor

13. $6x^2 + 5x - 21$ $(3x + 7)(2x - 3)$

14. $2a^2 + 14ab + 5ac + 35bc$ $(a + 7b)(2a + 5c)$

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| | 15. Absolute value, the penalty of 15 yards is the absolute value. |
| | 16. The application of the associative property makes this problem easier. This is structure! |
| | 17. This problem engages students in problem solving and fractions. Students typically apply the strategy of working backwards. |
| | 18-20. Part of a knowledge of the structure of algebra is realizing that not all trinomials can be factored. |
| | 21. Part of structure is realizing an ordered pair can be a solution and that there are typically an infinite number of solutions. |
| | Ask: Can anyone give other ordered pairs which satisfy the equation?<br>Ask: How many ordered pairs will satisfy the equation? |

15. The football team has the ball on the 29 yard line. There is a 15 yard penalty. Wherecould the ball be spotted after the penalty?

14 yard line or 43 yard line

16. Solve the following problem mentally:

(867 x 25) x 4 =___86,700_____

17. Rhonda cannot remember how much money she had when she went to the mall. She spent half of the money on a shirt and then spent two-thirds of what was left to buy a gift.If she now has $1.35, how much did Rhonda begin with?

$8.10

Factor each trinomial if you can, one cannot be factored.

18. $x^2 + 13x + 36$ 19. $x^2 - 5x - 7$ 20. $42x^2 + 11x - 3$

$(x + 9)(x + 4)$ cannot be factored $(6x - 1)(7x + 3)$

21. Which ordered pairs are solutions of the equation

$3x + 2y = 36$a. (4, 12) b. (8, 6) c. (5, 10)

Solution Solution Not a solution

*Instructor Notes TWA A11*

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| | Consider these four problems a set, specifically looking at what is the constant and the coefficient of the variable; what each of these mean; and more importantly how are each ofthem represented in the figures.<br><br>These are all linear generalizations. |
| | One of the most difficulty aspects of these problems is that students typically readily findthe recursive pattern, for example in #1, the pattern increases by 4 each time or +4.<br>However, many inevitably think the rule is $n + 4$ ’ I ask them what are they doing each time and they respond adding 4 and therefore they are repeatedly adding 4, sometimes I ask what is repeated addition and most know that it is multiplication, therefore the rule is not +4 but rather involves 4 times the number or 4n. Of course this is not the final rule, student can pick any value in the table and I ask how can you go across the table using the<br>4n, this is the explicit pattern and algebra! |
| | It can be challenging to help students to generalize without telling them the answer but it isworth it. Students must construct the mathematics themselves they cannot intuit it from our telling! Some have argued that generalization and symbolization are what constitutes algebraic thinking, thinking about algebra, this project goes a step further and is ‘thinking with algebra’! |
| | These problems are more algebra readiness and are designed to help student think about several key concepts: generalization, symbolic representation and the concept of variable. |
| | Talk about what the concept variable means? What does the ‘n’ represent in these expressions?What does generalization mean? |
| | Tables are important. Tables can help students focus on and find both the recursive pattern—down the table, and the explicit pattern—across the table. Research has shown that tables help students see mathematics, in this case make generalizations that they would not see without the table. Encourage students to find the relationship between the two numbers going across the table --the explicit pattern, they are exhibiting “**functionalthinking specifically a correspondence perspective**” (Stephens, Blanton, & Brizuela,<br>2017. p. 398). This is one approach to develop algebraic reasoning. |
| | Visual patterns are a very good way to introduce students to generalization and algebraic reasoning. Students should be encouraged to describe patterns visually—how does the pattern grow and numerically—the relationship between the numbers across the table, (the explicit pattern). In addition, the authors Stephens, Blanton, & Brizuela, (2017) emphasizethe importance of the social interactions in the classrooms and the norms that areestablished in developing algebra readiness. |